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Unsteady laminar compressible boundary-layer flow at a three-dimensional stagnation point

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Unsteady laminar compressible boundary-layer flow with variable properties at a three-dimensional stagnation point for both cold and hot walls has been studied for the case when the velocity of the incident stream varies arbitrarily with time. The partial differential equations governing the flow have been solved numerically using an implicit finite-difference scheme. Computations have been carried out for two particular unsteady free-stream velocity distributions: (i) an accelerating stream and (ii) a fluctuating stream. The results indicate that the variation of the density–viscosity product across the boundary layer, the wall temperature and the nature of stagnation point significantly affect the skin friction and heat transfer.

1. Introduction

The theory of the response of laminar boundary layers to variation of the external stream with time has many practical applications. It is of considerable interest in missile aerodynamics, in aircraft response to atmospheric gusts, in turbomachines and in flutter phenomena involving wings. Extensive reviews of the literature on the response of the laminar boundary layer to a fluctuating free stream and the literature on allied unsteady flows have been given by Stuart (1971), Riley (1975) and Telionis (1975). One of the best known studies is that of Lighthill (1954), who investigated the response of an incompressible laminar boundary layer over an arbitrary cylinder to small fluctuations in the external stream. Low and high frequency solutions were obtained by a momentum-integral method. Moore (1951), Moore & Ostrach (1956), Illingworth (1958), Gribben (1961) and King (1966) studied the unsteady laminar compressible boundary-layer flow over two-dimensional bodies by momentum-integral or series-expansion methods. Recently, Telionis & Gupta (1977) investigated the response of the compressible laminar boundary layer to small fluctuations in the outer flow under more general conditions for both two-dimensional and axisymmetric bodies. Solutions were presented for small amplitudes in the form of asymptotic expansions in powers of a frequency parameter. Gribben (1971) studied the compressible oscillating laminar boundary-layer flow in the neighbourhood of a two-dimensional stagnation point on a hot surface. Solutions were given as series valid for low-and high frequencies. Vimala & Nath (1975) studied the above problem as well as a constantly accelerating flow problem for a cold wall and solved the governing partial differential equations numerically using an implicit finite-difference scheme. The problem was tackled as an initial-value problem, starting from a steady solution, and hence the solution was different from other work (Lighthill 1954; Gribben 1961, 1971), where transient motions were assumed to have died away. It may be noted that all the above authors assumed the product of density and viscosity to be constant across the boundary layer. This simplification leads to appreciable errors in the analysis. Moreover, they considered unsteady flow over two-dimensional and axi-symmetric bodies only. It is well known that unsteady flows over three-dimensional bodies are of more practical interest.

In this investigation, we have studied the unsteady laminar compressible boundary-layer flow in the immediate neighbourhood of the stagnation points on a class of three-dimensional bodies ranging from spheres through cylinders to saddle shapes. The magnitudes of the inviscid velocity gradients in the two principal planes have been taken to be equal and account has been taken of the effect of the variation of the density–viscosity product across the boundary layer. Both cold and hot walls have been considered. The unsteadiness in the present case is due to arbitrary variations in the velocity of the incident stream with time. The problem has been solved as an initial-value problem, starting from a steady solution (Vimala & Nath 1975). The partial differential equations governing the flow have been solved numerically using an implicit finite-difference scheme after transformation to new co-ordinates with a finite domain (Marvin & Sheaffer 1969; Vimala & Nath 1975). Computations have been carried out for the following free-stream velocity distributions: (i) a stream moving with constant acceleration and (ii) a stream fluctuating about a steady mean. The steady-state results have been compared with those obtained by Libby (1967) and Nath & Muthanna (1977) and the unsteady-state results with those of Vimala & Nath (1975).

2. Governing equations

If it is assumed that the external flow is homentropic, that the surface is maintained at a constant temperature and that the dissipation terms are negligible at the stagnation point, then the boundary-layer equations governing unsteady laminar compressible flow at the stagnation point of a three-dimensional body can be expressed as (Libby 1967)

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0, \tag{2.1a}$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=\rho_e\left(\frac{\partial u_e}{\partial t}+u_e\frac{\partial u_e}{\partial x}\right)+\frac{\partial}{\partial z}\left(\mu\frac{\partial u}{\partial z}\right), \tag{2.1b}$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=\rho_e\left(\frac{\partial v_e}{\partial t}+v_e\frac{\partial v_e}{\partial y}\right)+\frac{\partial}{\partial z}\left(\mu\frac{\partial v}{\partial z}\right), \tag{2.1c}$$

$$\rho\left(\frac{\partial h}{\partial t}+u\frac{\partial h}{\partial x}+v\frac{\partial h}{\partial y}+w\frac{\partial h}{\partial z}\right)=\frac{1}{\sigma}\frac{\partial}{\partial z}\left(\mu\frac{\partial h}{\partial z}\right). \tag{2.1d}$$

Here t is the time, x, y and z are the principal, transverse and normal directions, respectively, u, v and w are the velocity components in the x, y and z directions, respectively, and ρ, μ, h and σ are, respectively, the density, viscosity, specific enthalpy and Prandtl number. The suffix e denotes free-stream values.

The relevant initial and boundary conditions are

$$\left.\begin{aligned} u(x,y,z,0) &= u_i(x,y,z), \quad v(x,y,z,0) = v_i(x,y,z),\\ w(x,y,z,0) &= w_i(x,y,z), \quad h(x,y,z,0) = h_i(x,y,z),\end{aligned}\right\} \tag{2.2a}$$

$$\left.\begin{aligned} u(x,y,0,t) &= v(x,y,0,t) = w(x,y,0,t) = 0,\\ h(x,y,0,t) &= h_w,\end{aligned}\right\} \tag{2.2b}$$

$$\left.\begin{aligned} u(x,y,\infty,t) &= u_e(x,t), \quad v(x,y,\infty,t) = v_e(y,t),\\ h(x,y,\infty,t) &= h_e,\end{aligned}\right\} \tag{2.2c}$$

where the suffixes i and w denote values at the initial time $t = 0$ and at the wall $z = 0$, respectively, and h_w and h_e are constants. For three-dimensional stagnation-point flow, free-stream velocities u_e and v_e which vary arbitrarily with time can, without loss of generality, be expressed in the form

$$u_e = ax\phi(t^*), \quad v_e = by\phi(t^*), \quad t^* = at, \tag{2.2d}$$

where $a = (du_e/dx)_0$ and $b = (dv_e/dy)_0$ are constants, a has the dimensions $(\text{time})^{-1}$, a suffix zero denotes the value at the stagnation point and ϕ is an arbitrary function representing the nature of the unsteadiness in the external stream and has a continuous first derivative for $t^* > 0$.

On applying the transformations

$$\left.\begin{aligned} \eta &= (\rho_e a/\mu_e)^{\frac{1}{2}} \int_0^z (\rho/\rho_e)\, dz,\\ u &= ax\phi(t^*)\, \partial f(\eta,t^*)/\partial\eta, \quad v = by\phi(t^*)\, \partial s(\eta,t^*)/\partial\eta,\\ w &= -(\rho_e/\rho)\,(\mu_e a/\rho_e)^{\frac{1}{2}}\,[\phi(t^*)\,(f+cs) + \partial\eta/\partial t^*],\end{aligned}\right\} \tag{2.3a}$$

$$h = h_e g(\eta,t^*), \quad c = b/a, \quad F = \partial f/\partial\eta, \quad S = \partial s/\partial\eta \tag{2.3b}$$

to (2.1), we find that (2.1 a) is satisfied identically and (2.1 b–d) reduce to

$$\frac{\partial}{\partial\eta}\left(N\frac{\partial F}{\partial\eta}\right) + \phi\left[(f+cs)\frac{\partial F}{\partial\eta} + g - F^2\right] + \frac{1}{\phi}\frac{d\phi}{dt^*}(g-F) - \frac{\partial F}{\partial t^*} = 0, \tag{2.4a}$$

$$\frac{\partial}{\partial\eta}\left(N\frac{\partial S}{\partial\eta}\right) + \phi\left[(f+cs)\frac{\partial S}{\partial\eta} + c(g - S^2)\right] + \frac{1}{\phi}\frac{d\phi}{dt^*}(g-S) - \frac{\partial S}{\partial t^*} = 0, \tag{2.4b}$$

$$\frac{1}{\sigma}\frac{\partial}{\partial\eta}\left(N\frac{\partial g}{\partial\eta}\right) + \phi(f+cs)\frac{\partial g}{\partial\eta} - \frac{\partial g}{\partial t^*} = 0, \tag{2.4c}$$

where
$$N = \rho\mu/\rho_e\mu_e = g^{\omega-1}, \quad \rho_e/\rho = h/h_e, \quad \mu/\mu_e = (h/h_e)^{\omega}, \tag{2.5}$$

N being the ratio of the density–viscosity products in the boundary layer and the free stream. The transformed boundary conditions are given by

$$\left.\begin{aligned} F &= S = 0, \quad g = g_w \quad \text{at} \quad \eta = 0\\ F &\to 1, \quad S \to 1, \quad g \to 1 \quad \text{as} \quad \eta \to \infty\end{aligned}\right\} \text{ at time } t^*. \tag{2.6}$$

We assume that the flow is initially steady then becomes unsteady for $t^* > 0$. Hence the initial conditions for $F(\eta,t^*)$, $S(\eta,t^*)$ and $g(\eta,t^*)$ at $t^* = 0$ are given by the steady-flow equations obtained by putting

$$\phi(t^*) = 1, \quad d\phi/dt^* = 0, \quad \partial/\partial t^* = 0 \tag{2.7}$$

in (2.4). These steady-state equations are the same as those obtained by Libby (1967). In (2.4), $c = 0$ corresponds to two-dimensional stagnation-point flow, which has been studied by Vimala & Nath (1975) for $N = 1$, while $c = 1$ represents axisymmetric stagnation-point flow. The aim of the present study is to obtain solutions for both nodal ($0 \leqslant c \leqslant 1$) and saddle ($-1 \leqslant c < 0$) points of attachment taking into account the variation of N across the boundary layer. It may be noted that $\omega = 0{\cdot}5$ corresponds to the conditions encountered in hypersonic flight, $\omega = 0{\cdot}7$ corresponds to low-temperature flows and $\omega = 1$ represents the simplification of a constant density–viscosity product (Gross & Dewey 1965).

The skin-friction coefficients along the x and y directions and the heat-transfer coefficient, in the form of the Stanton number St, can be expressed as (Libby 1967; Nath & Muthanna 1977)

$$C_f = 2\tau_x/\rho_e(u_e^2)_{t^*=0} = 2Re_x^{-\frac{1}{2}} g_w^{\omega-1}\phi(t^*)(\partial F/\partial\eta)_w, \tag{2.8a}$$

$$\bar{C}_f = 2\tau_y/\rho_e(u_e^2)_{t^*=0} = 2Re_x^{-\frac{1}{2}}(v_e/u_e) g_w^{\omega-1}\phi(t^*)(\partial S/\partial\eta)_w, \tag{2.8b}$$

$$St = q_w/[(h_e-h_w)\rho_e(u_e)_{t^*=0}] = Re_x^{-\frac{1}{2}}\sigma^{-1}(1-g_w)^{-1}g_w^{\omega-1}(\partial g/\partial\eta)_w, \tag{2.8c}$$

where
$$\tau_x = \mu_w(\partial u/\partial z)_w, \quad \tau_y = \mu_w(\partial v/\partial z)_w, \tag{2.9a}$$

$$q_w = \mu_w\sigma^{-1}(\partial h/\partial z)_w, \quad Re_x = ax^2/\nu_e. \tag{2.9b}$$

Here τ_x and τ_y are the wall shear stresses in the x and y directions, respectively; C_f and $\bar{C}_f$ are the skin-friction coefficients in the x and y directions, respectively; q_w is the rate of heat transfer; Re_x is the Reynolds number; ν is the kinematic viscosity; the velocity gradients $(\partial F/\partial\eta)_w$ and $(\partial S/\partial\eta)_w$ at the wall represent skin-friction parameters in the x and y directions, respectively; and the enthalpy gradient $(\partial g/\partial\eta)_w$ at the wall is the heat-transfer parameter.

3. Transformation to finite co-ordinates

We use the transformations

$$Y = 1-\exp(-\alpha\eta), \quad Z = \alpha(1-Y) \tag{3.1}$$

to transform the system of partial differential equations (2.4) to a new system of co-ordinates wherein the range of variation $(0, \infty)$ for η is replaced by the range $(0, 1)$ for Y. Here the constant α is a scaling factor chosen to provide an optimum distribution at nodal points across the boundary layer. The system of equations (2.4) can now be expressed as

$$Z^2N\frac{\partial^2 F}{\partial Y^2}+Z\left[Z\frac{\partial N}{\partial Y}-N\alpha+\phi(f+cs)\right]\frac{\partial F}{\partial Y}+\phi(g-F^2)+\phi^{-1}\frac{d\phi}{dt^*}(g-F)-\frac{\partial F}{\partial t^*} = 0, \tag{3.2a}$$

$$Z^2N\frac{\partial^2 S}{\partial Y^2}+Z\left[Z\frac{\partial N}{\partial Y}-N\alpha+\phi(f+cs)\right]\frac{\partial S}{\partial Y}+\phi c(g-S^2)+\phi^{-1}\frac{d\phi}{dt^*}(g-S)-\frac{\partial S}{\partial t^*} = 0, \tag{3.2b}$$

$$\sigma^{-1}Z^2N\frac{\partial^2 g}{\partial Y^2}+Z\left[\sigma^{-1}\left(Z\frac{\partial N}{\partial Y}-N\alpha\right)+\phi(f+cs)\right]\frac{\partial g}{\partial Y}-\frac{\partial g}{\partial t^*} = 0. \tag{3.2c}$$

The initial conditions (at $t^* = 0$) are

$$Z^2N\frac{\partial^2 F}{\partial Y^2}+Z\left[Z\frac{\partial N}{\partial Y}-N\alpha+f+cs\right]\frac{\partial F}{\partial Y}+g-F^2 = 0, \tag{3.3a}$$

$$Z^2N\frac{\partial^2 S}{\partial Y^2}+Z\left[Z\frac{\partial N}{\partial Y}-N\alpha+f+cs\right]\frac{\partial S}{\partial Y}+c(g-S^2) = 0, \tag{3.3b}$$

$$\sigma^{-1}Z^2N\frac{\partial^2 g}{\partial Y^2}+Z\left[\sigma^{-1}\left(Z\frac{\partial N}{\partial Y}-N\alpha\right)+f+cs\right]\frac{\partial g}{\partial Y} = 0. \tag{3.3c}$$

The boundary conditions are

$$\left.\begin{array}{l} F = S = 0, \quad g = g_w \quad \text{at} \quad Y = 0 \\ F = S = g = 1 \quad \text{at} \quad Y = 1 \end{array}\right\} \quad \text{for} \quad t^* \geqslant 0. \tag{3.4}$$

Here f and s are given by

$$f = \int_0^Y (F/Z)\,dY, \quad s = \int_0^Y (S/Z)\,dY. \tag{3.5}$$

The skin-friction and heat-transfer parameters can now be written as

$$\left(\frac{\partial F}{\partial \eta}\right)_w = \alpha\left(\frac{\partial F}{\partial Y}\right)_w, \quad \left(\frac{\partial S}{\partial \eta}\right)_w = \alpha\left(\frac{\partial S}{\partial Y}\right)_w, \quad \left(\frac{\partial g}{\partial \eta}\right)_w = \alpha\left(\frac{\partial g}{\partial Y}\right)_w. \tag{3.6}$$

4. Results and discussion

The set of equations (3.2) has been solved numerically under conditions (3.3) and (3.4) using an implicit finite-difference scheme. Since the method is described in great detail by Marvin & Sheaffer (1969) and Vimala & Nath (1975), the description is not repeated here. Computations have been carried out for various values of the parameters c $(-1 \leqslant c \leqslant 1)$, ω $(0{\cdot}5 \leqslant \omega \leqslant 1{\cdot}0)$ and g_w $(0{\cdot}25 \leqslant g_w \leqslant 1{\cdot}5)$ with $\sigma = 0{\cdot}72$, $\alpha = 0{\cdot}5$, $\epsilon = 0{\cdot}1$, $\omega^* = 5{\cdot}6$, $\Delta Y = 0{\cdot}05$ and $\Delta t^* = 0{\cdot}1$, but for the sake of brevity only results for some of the values of the parameters are presented here.† The unsteady free-stream velocity distributions considered here are given by

$$\phi(t^*) = 1+t^*, \quad \phi(t^*) = 1+\epsilon\sin(\omega^* t^*),$$

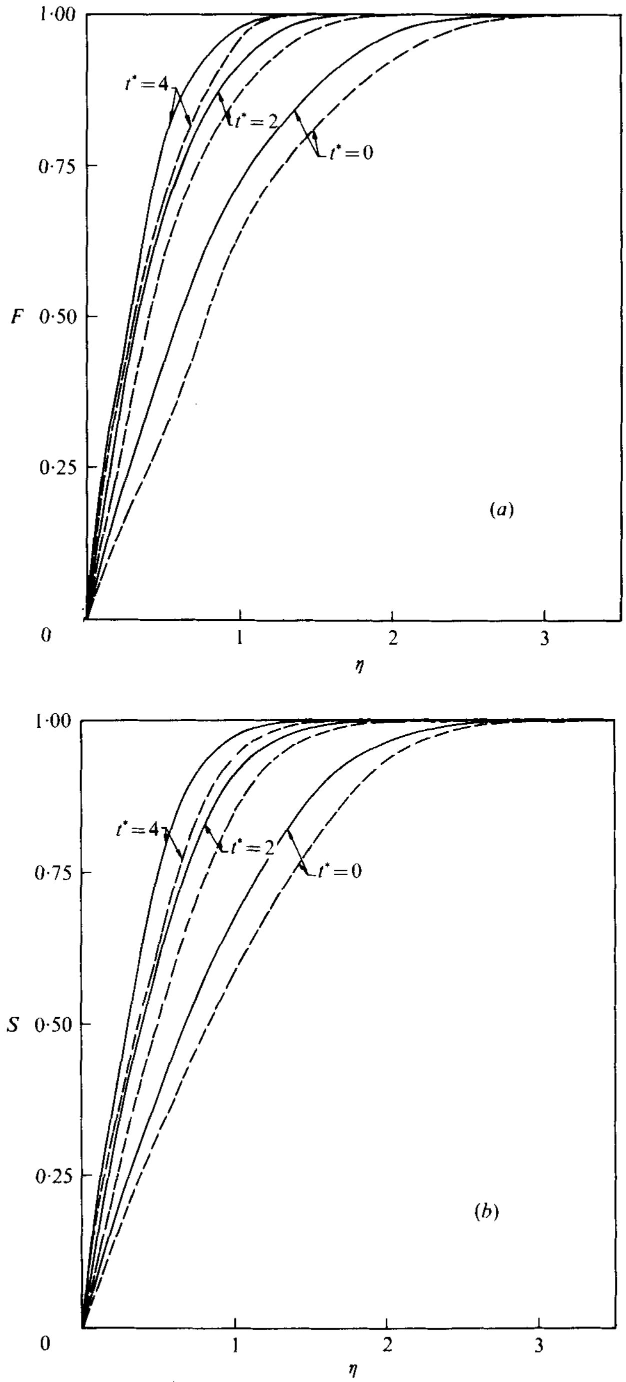
where ϵ is a small constant and ω^* is the frequency parameter. Further reduction in the step size ΔY and Δt^* changes the results only in the fourth decimal place. The results for the case $\phi(t^*) = 1+t^*$ are given in figures 1–3 and those for the case

$$\phi(t^*) = 1+\epsilon\sin(\omega^* t^*)$$

in figures 4 and 5.

Some representative velocity and enthalpy profiles F, S and g for a cold wall $(g_w < 1)$ are shown in figure 1. These profiles become steeper as the dimensionless time t^*, the index ω of the power-law viscosity variation or the parameter c $(\geqslant 0)$ characterizing the nature of the stagnation point increases. When $c < 0$ and $g_w < 1$,

† The results for other values of the parameters may be obtained from the authors.



FIGURES 1 (a, b). For legend see facing page.

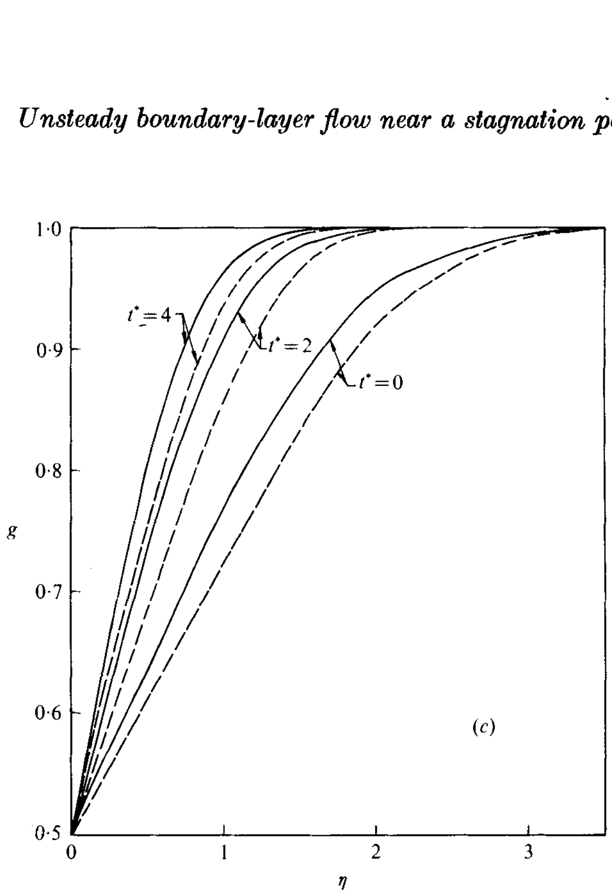
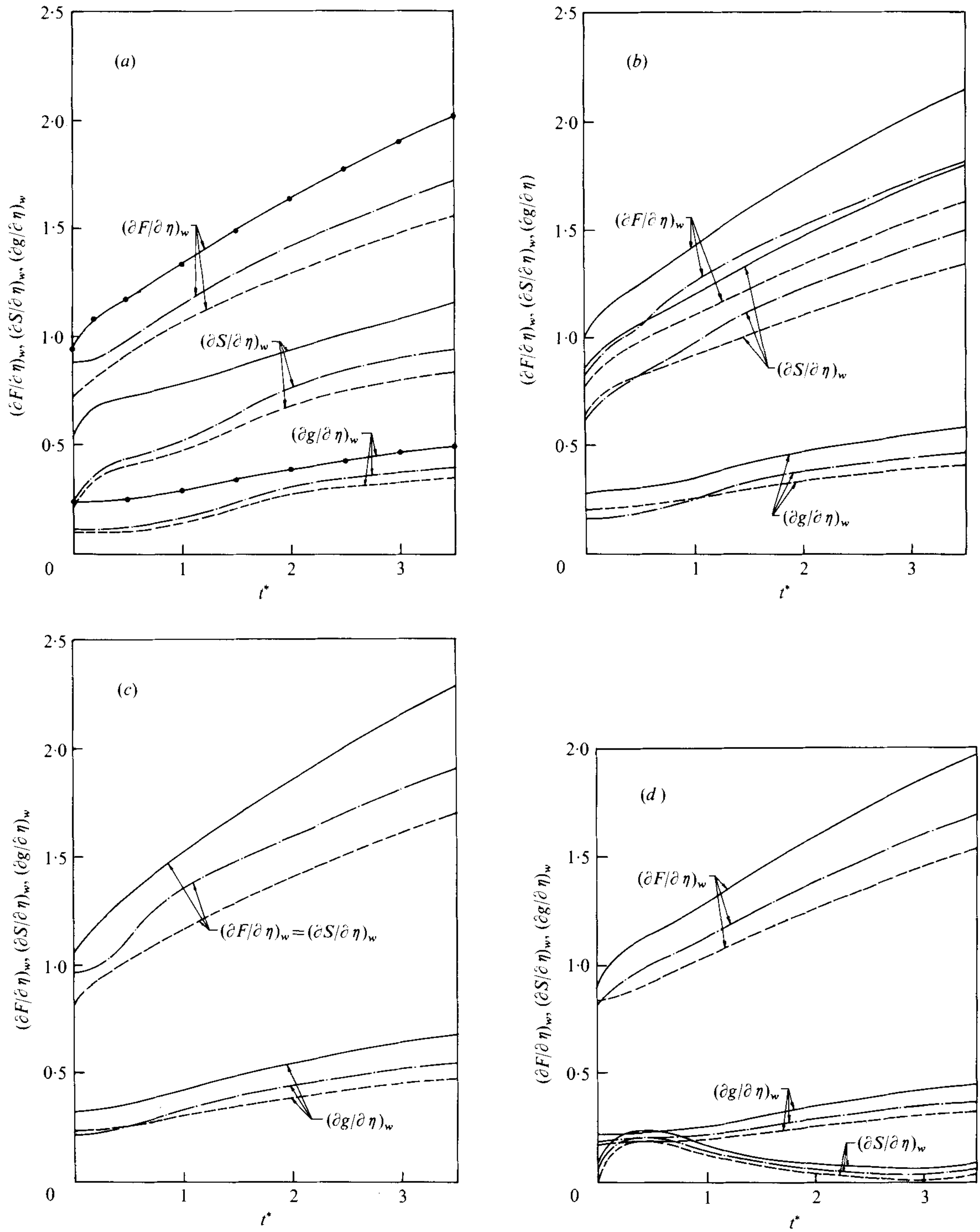


FIGURE 1. (*a*) Velocity profiles in the x direction, (*b*) velocity profiles in the y direction and (*c*) enthalpy profiles for $t^* = 0$, 2 and 4, $c = 0{\cdot}5$, $\phi(t^*) = 1+t^*$ ($t^* \geqslant 0$) and $g_w = 0{\cdot}5$. ——, $\omega = 1{\cdot}0$; ‒ ‒ ‒, $\omega = 0{\cdot}5$.

the F, S and g profiles become less steep when c decreases through the range $c^* \leqslant c < 0$, but in the range $-1 \leqslant c < c^*$ they become steeper as c decreases from c^*. A similar trend has been observed by Libby (1967) and Nath & Muthanna (1977) for the steady-state case ($t^* = 0$). The profiles showing the effect of c and g_w are not presented here for the sake of brevity. The skin-friction parameters $(\partial F/\partial\eta)_w$ and $(\partial S/\partial\eta)_w$ in the principal and transverse directions and the heat-transfer parameter $(\partial g/\partial\eta)_w$ for the cold-wall case ($g_w < 1$) are shown in figures 2(*a*)–(*e*). It is evident from these figures that, for $c \geqslant 0$ (nodal points of attachment) and $g_w < 1$, $(\partial F/\partial\eta)_w$, $(\partial S/\partial\eta)_w$ and $(\partial g/\partial\eta)_w$ increase with t^* (in some cases after a certain time) whatever the value of ω. When $c < 0$ (saddle points of attachment) and $g_w < 1$, $(\partial F/\partial\eta)_w$ and $(\partial g/\partial\eta)_w$ still increase with t^* but $(\partial S/\partial\eta)_w$ first increases then decreases as t^* increases. For a given t^*, $(\partial F/\partial\eta)_w$, $(\partial S/\partial\eta)_w$ and $(\partial g/\partial\eta)_w$ increase as ω or g_w increases except that $(\partial g/\partial\eta)_w$ decreases as g_w increases. It is seen that, for all values of ω, g_w and t^*, $(\partial F/\partial\eta)_w$, $(\partial S/\partial\eta)_w$ and $(\partial g/\partial\eta)_w$ decrease as c decreases until at some negative c the parameter $(\partial S/\partial\eta)_w$ is reversed and $(\partial F/\partial\eta)_w$ and $(\partial g/\partial\eta)_w$ begin to increase as c decreases. This trend has also been observed by Libby (1967) and Nath & Muthanna (1977) for the steady-state case.

The results for $g_w = 1{\cdot}5$ (hot wall) are given in figures 3(*a*) and (*b*). It is seen from these figures that $(\partial F/\partial\eta)_w$ increases but $(\partial g/\partial\eta)_w$ (which is negative) decreases as t^* increases. This is true for all values of c and ω. However, for $c \geqslant 0$, $(\partial S/\partial\eta)_w$ increases with t^*, but for $c < 0$ it first increases then decreases. For a hot wall ($g_w > 1$), $(\partial F/\partial\eta)_w$



FIGURES 2 (*a–d*). For legend see facing page.

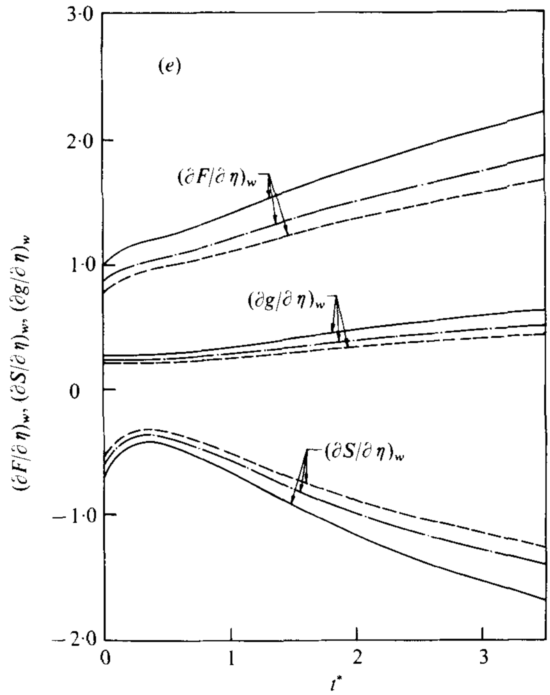


FIGURE 2. Skin-friction and heat-transfer parameters for (*a*) $c = 0$, (*b*) $c = 0{\cdot}5$, (*c*) $c = 1{\cdot}0$, (*d*) $c = -0{\cdot}5$ and (*e*) $c = -1{\cdot}0$. $\phi(t^*) = 1+t^*$ for $t^* \geqslant 0$; $g_w = 0{\cdot}5$. ——, $\omega = 1{\cdot}0$; -·-, $\omega = 0{\cdot}7$; ---, $\omega = 0{\cdot}5$; ●, Vimala & Nath.

and $(\partial S/\partial\eta)_w$ increase as ω decreases in contrast with the cold-wall case ($g_w < 1$). However, like the cold-wall case, $(\partial g/\partial\eta)_w$ decreases as ω decreases.

Some representative skin-friction and heat-transfer results for the oscillatory free-stream velocity distribution $\phi(t^*) = 1+\epsilon\sin(\omega^*t^*)$ are shown in figures 4 and 5. It is clear from these figures that the skin-friction parameters $(\partial F/\partial\eta)_w$ and $(\partial S/\partial\eta)_w$ respond more to the fluctuations in the free-stream velocity than does the heat-transfer parameter $(\partial g/\partial\eta)_w$. This behaviour holds whatever the values of ω, c and g_w. The effect of ω on $(\partial F/\partial\eta)_w$, $(\partial S/\partial\eta)_w$ and $(\partial g/\partial\eta)_w$ is more pronounced when $c > 0$ than when $c < 0$.

The skin-friction and heat-transfer results for the steady-state case ($t^* = 0$) have been compared with those of Libby (1967) and Nath & Muthanna (1977) and found to be in excellent agreement. Furthermore, we have also compared the skin-friction and heat-transfer results for the unsteady case ($t^* > 0$) for $c = 0$ (two-dimensional case), $\omega = 1$ and $g_w = 0{\cdot}5$ with those of Vimala & Nath (1975) and found excellent agreement. It may be remarked that the present analysis is more general than those performed so far.

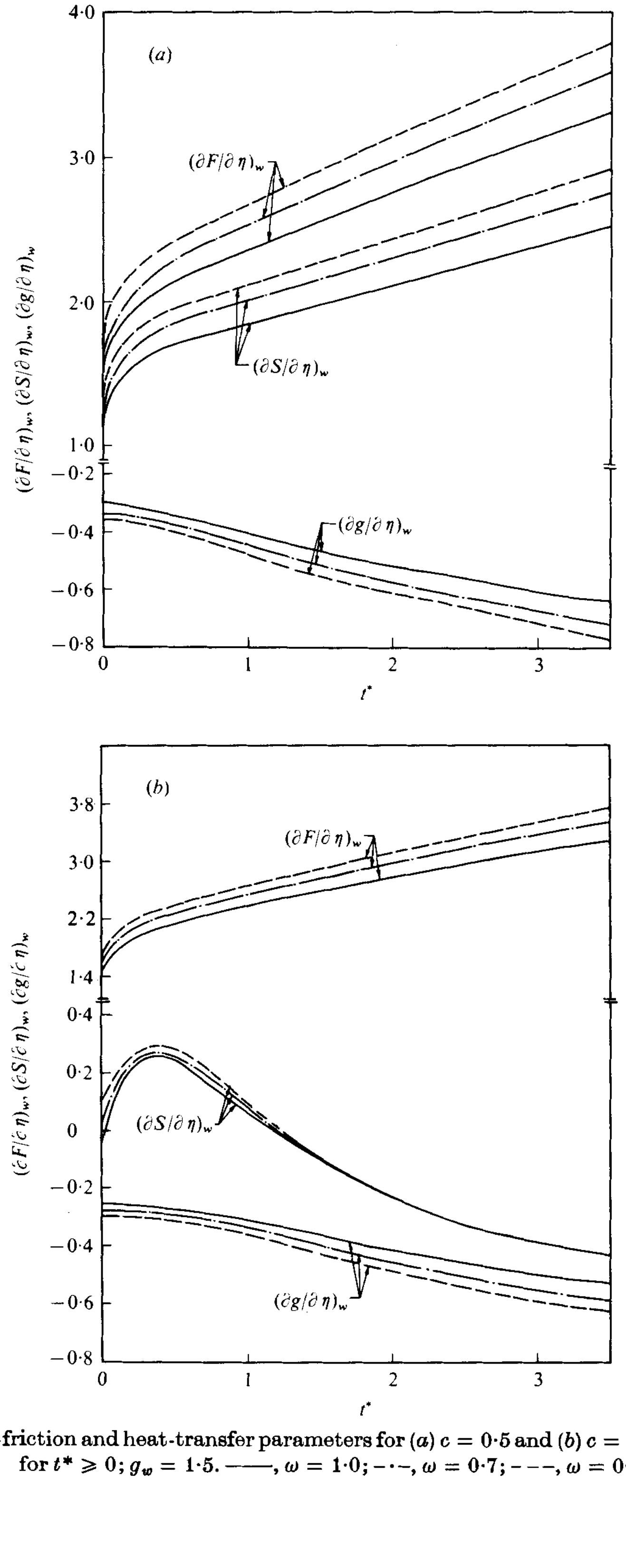


FIGURE 3. Skin-friction and heat-transfer parameters for (a) $c = 0{\cdot}5$ and (b) $c = -0{\cdot}5$. $\phi(t^*) = 1+t^*$ for $t^* \geqslant 0$; $g_w = 1{\cdot}5$. ——, $\omega = 1{\cdot}0$; $-\cdot-$, $\omega = 0{\cdot}7$; $---$, $\omega = 0{\cdot}5$.

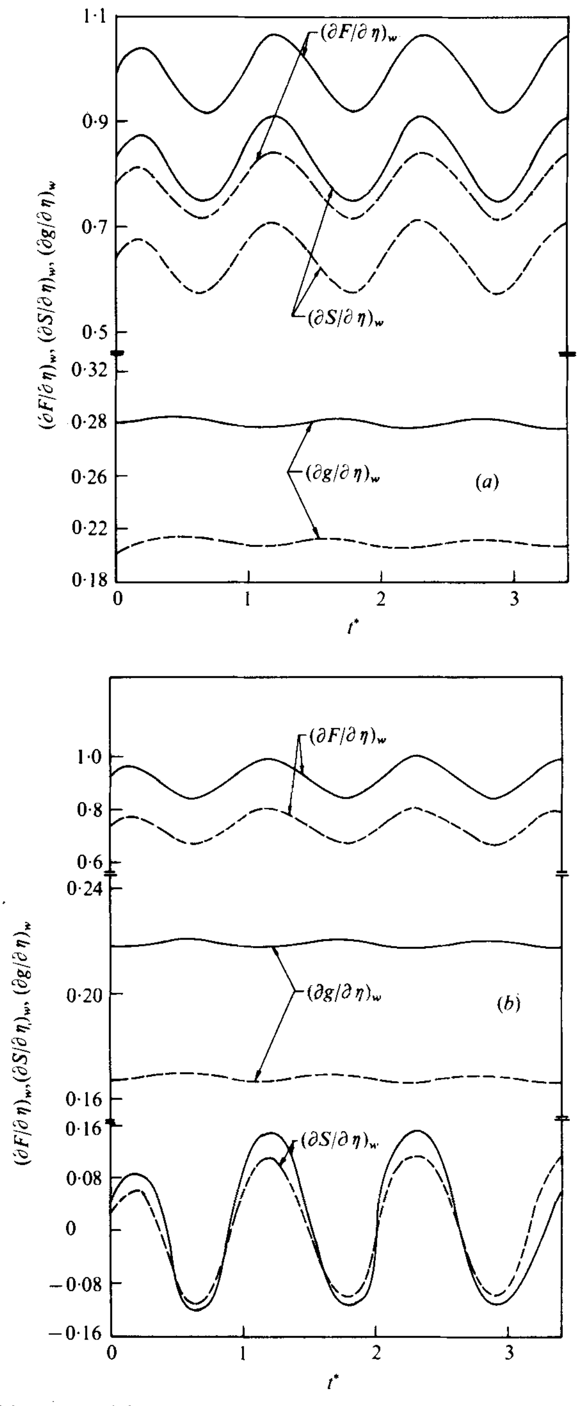
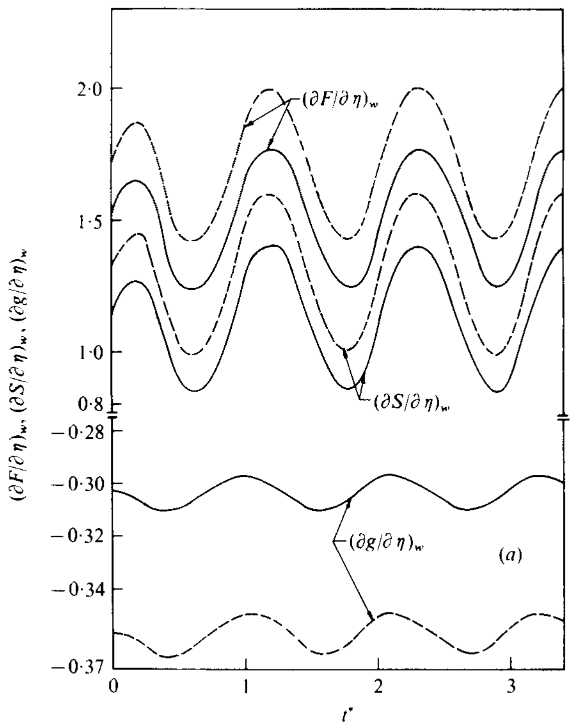


FIGURE 4. Skin-friction and heat-transfer parameters for (*a*) $c = 0{\cdot}5$ and (*b*) $c = -0{\cdot}5$. $\phi(t^*) = 1 + \epsilon \sin(\omega^* t^*)$ for $t^* \geqslant 0$; $g_w = 0{\cdot}5$; $\epsilon = 0{\cdot}1$; $\omega^* = 5{\cdot}6$. ——, $\omega = 1{\cdot}0$; – – –, $\omega = 0{\cdot}5$.



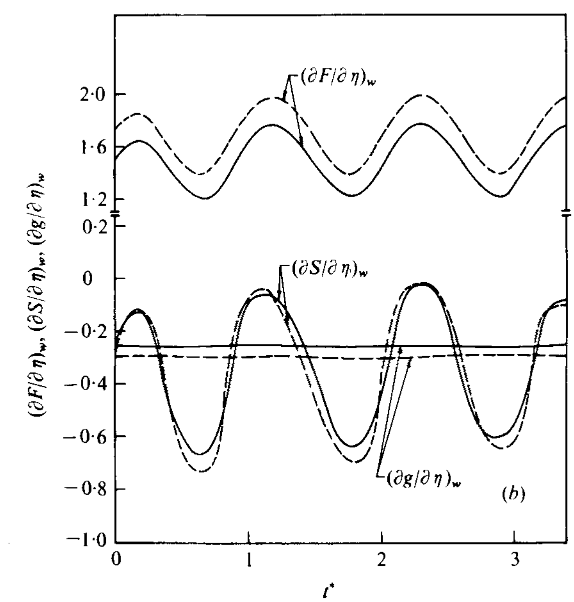


FIGURE 5. Skin-friction and heat-transfer parameters for (*a*) $c = 0{\cdot}5$ and (*b*) $c = -0{\cdot}5$. $\phi(t^*) = 1+\epsilon \sin(\omega^* t^*)$ for $t^* \geqslant 0$; $g_w = 1{\cdot}5$; $\epsilon = 0{\cdot}1$; $\omega^* = 5{\cdot}6$. ——, $\omega = 1{\cdot}0$; – – –, $\omega = 0{\cdot}5$.

5. Conclusions

The results show that the variation of the density–viscosity product across the boundary layer, the nature of the stagnation point and the wall temperature exert a strong influence on the skin-friction and heat-transfer parameters. Furthermore, the skin-friction parameter responds more to the fluctuations in the free-stream oscillating velocities than does the heat-transfer parameter.

REFERENCES

GRIBBEN, R. J. 1961 The laminar boundary layer on a hot cylinder fixed in a fluctuating stream. *Trans. A.S.M.E., J. Appl. Mech.* **28**, 339.

GRIBBEN, R. J. 1971 The fluctuating flow of a gas near a stagnation point on a hot wall. *Trans. A.S.M.E., J. Appl. Mech.* **38**, 820.

GROSS, J. F. & DEWEY, C. F. 1965 Similar solutions of the laminar boundary layer equations with variable fluid properties. In *Fluid Dyn. Trans.* (ed. W. Fiszdon), vol. 2, p. 529. Pergamon.

ILLINGWORTH, C. R. 1958 The effects of a sound wave on the compressible boundary layer on a flat plate. *J. Fluid Mech.* **3**, 471.

KING, W. S. 1966 Low frequency, large amplitude fluctuations of the laminar boundary layer. *A.I.A.A. J.* **4**, 994.

LIBBY, P. A. 1967 Heat and mass transfer at a general three-dimensional stagnation point. *A.I.A.A. J.* **5**, 507.

LIGHTHILL, M. J. 1954 The response of laminar skin friction and heat transfer to fluctuations in the stream velocity. *Proc. Roy. Soc.* A **224**, 1.

MARVIN, J. G. & SHEAFFER, Y. S. 1969 A method for solving nonsimilar laminar boundary layer equations including foreign gas injection. *N.A.S.A. Tech. Note* D5516.

MOORE, F. K. 1951 Unsteady laminar boundary layer flow. *N.A.C.A. Tech. Note* no. 2471.

MOORE, F. K. & OSTRACH, S. 1956 Average properties of a compressible laminar boundary layer on a flat plate with unsteady flight velocity. *N.A.C.A. Tech. Note* no. 3886.

NATH, G. & MUTHANNA, M. 1977 Laminar hypersonic boundary-layer flow at a three-dimensional stagnation point with slip and mass transfer. *Int. J. Heat Mass Transfer* **20**, 1977.

RILEY, N. 1975 Unsteady laminar boundary layers. *SIAM Rev.* **17**, 274.

STUART, J. T. 1972 Unsteady boundary layers. In *Recent Research on Unsteady Boundary Layers*, vol. 1, p. 1. Laval University Press, Quebec.

TELIONIS, D. P. 1975 Calculations of time dependent boundary layers. In *Unsteady Aerodynamics* (ed. R. B. Kinney), vol. 1, p. 155. University of Arizona, Tucson.

TELIONIS, D. P. & GUPTA, T. R. 1977 Compressible oscillating boundary layers. *A.I.A.A. J.* **15**, 974.

VIMALA, C. S. & NATH, G. 1975 Unsteady laminar boundary layers in a compressible stagnation flow. *J. Fluid Mech.* **70**, 561.